Tanta University
Faculty of Engineering
Dept. of Computers and Control

Second Year Students
Year 2015 / 2016
Second Semester

SIGNALS and SYSTEMS Code: CCE 2210

(53) The characteristic polynomial of a continuous-time system is given below. Locate the roots of the characteristic equation in the s-plane, and hence determine whether the system is stable, marginally stable, or unstable.

(a) $q(s) = (s+1)(s^2 + 8s + 12)$

(b) $q(s) = (s+3)(s^2+1)$

(c) $q(s) = (s+4)(s^2-9)$

(d) $q(s) = (s^2+3s+2)(s^4+2s^2+1)$

(54) For each of the following characteristic equations, determine the root distribution, i.e. the number of roots in the left half, right half, and on the imaginary axis of the s-plane. Is the corresponding system stable, marginally stable, or unstable? Why!?

(a) $s^3 + 8s^2 + 19s + 12 = 0$

(b) $s^4 + 2s^3 + 3s^2 + 6s + 1 = 0$

(c) $s^4 + s^3 + 2s^2 + s + 3 = 0$

(d) $s^3 + 2s^2 + 4s + 8 = 0$

(e) $s^5 + 3s^4 + 2s^3 + 6s^2 + s + 3 = 0$

(f) $s^6 + s^5 + 3s^4 + 3s^3 + 3s^2 + 2s + 1 = 0$

(55) A unity feedback system has a forward transfer function

$$G(s) = \frac{K}{s(s+4)(s^2+s+1)}$$

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Find the range of K for which the system is stable.

(56) Consider the system shown in Fig. 24. By applying the Routh-Hurwitz criterion, discuss the stability of the system in terms of K. Determine the value of K which results in sustained oscillations. What is the corresponding frequency of oscillation ?

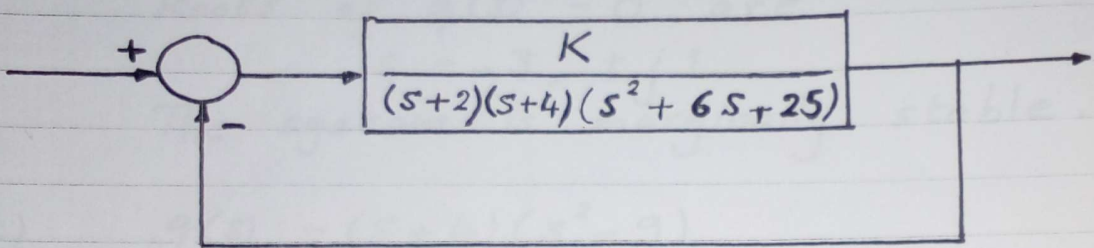
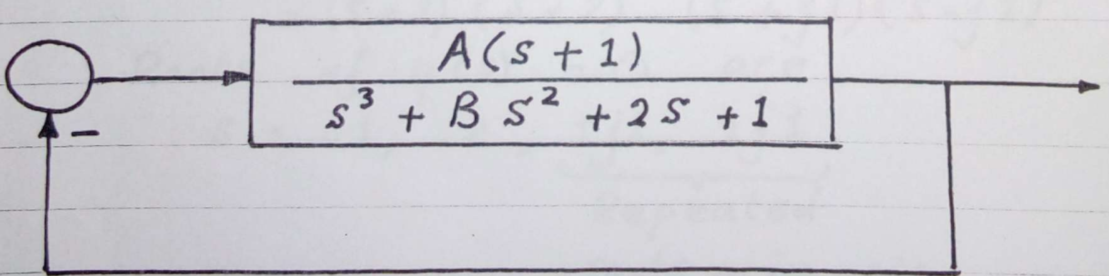


Fig. 24 System for Prob. 56

(57) An oscillator system is shown in Fig. 25. Determine the values of A and B such that the system oscillates at a frequency of 2 rad/sec.



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Problem 53

(a) $q(s) = (s+1)(s^2 + 8s + 12)$

$= (s+1)(s+2)(s+6)$

Roots of $q(s) = 0$ are

$s = -1, -2, -6$

The system is stable.

(b) $q(s) = (s+3)(s^2+1)$

$= (s+3)(s+j1)(s-j1)$

Roots of $q(s) = 0$ are

$s = -3, \pm j1$

The system is marginally stable.

(c) $q(s) = (s+4)(s^2-9)$

$= (s+4)(s+3)(s-3)$

Roots of $q(s) = 0$ are

$s = -4, -3, 3$

The system is unstable.

(d) $q(s) = (s^2 + 3s + 2)(s^4 + 2s^2 + 1)$

$= (s+1)(s+2)(s^2+1)^2$

$= (s+1)(s+2)(s+j1)^2(s-j1)^2$

Roots of $q(s) = 0$ are

$s = -1, -2, \underbrace{\pm j1, \pm j1}_{\text{Repeated roots on } j\omega\text{-axis}}$

The system is unstable.

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Problem 54

(a) $s^3 + 8s^2 + 19s + 12 = 0$

Routh array

$$
\begin{array}{cc|ccc}
+ & s^3 & 1 & 19 & 0 \\
+ & s^2 & 8 & 12 & 0 \\
+ & s^1 & \frac{35}{2} & 0 & \\
+ & s^0 & 12 & &
\end{array}
$$

No all-zero rows, $n_0 = 0$

No change in signs, $n_+ = 0$

$$
\begin{aligned}
n_- &= n - (n_+ + n_0) \\
&= 3 - (0+0) \\
&= 3
\end{aligned}
$$

Root distribution : $\{3(-),\ 0(0),\ 0(+)\}$

The system is stable.

(b) $s^4 + 2s^3 + 3s^2 + 6s + 1 = 0$

Routh array

$$
\begin{array}{cc|ccc}
+ & s^4 & 1 & 3 & 1 \\
 & \cancel{s^3} & \cancel{2} & \cancel{6} & \\
+ & s^3 & 1 & 3 & \\
 & \cancel{s^2} & \cancel{0} & \cancel{1} & \\
+ & s^2 & \epsilon & 1 & \\
- & s^1 & \frac{3\epsilon - 1}{\epsilon} & & \\
+ & s^0 & 1 & &
\end{array}
$$

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No all-zero rows, $n_0 = 0$
Number of sign changes $= 2$, $n_+ = 2$

$$
\begin{aligned}
n_- &= n - (n_+ + n_0) \\
&= 4 - (2 + 0) \\
&= 2
\end{aligned}
$$

Root distribution : $\{2(-),\ 0(0),\ 2(+)\}$
The system is unstable.

(c) $s^4 + s^3 + 2s^2 + s + 3 = 0$

Routh array

+	s^4	1	2	3
+	s^3	1	1	
+	s^2	1	3	
−	s^1	−2		
+	s^0	3		

No all-zero rows, $n_0 = 0$
Number of sign changes $= 2$, $n_+ = 2$

$$
\begin{aligned}
n_- &= n - (n_+ + n_0) = 4 - (2 + 0) \\
&= 2
\end{aligned}
$$

Root distribution : $\{2(-),\ 0(0),\ 2(+)\}$
The system is unstable.

(d) $s^3 + 2s^2 + 4s + 8 = 0$

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Routh array

+	s^3	1	4
	$\cancel{s^2}$	$\cancel{2}$	$\cancel{8}$
+	s^2	1	4
	$\cancel{s^1}$	$\cancel{0}$	
+	s^1	2	
+	s^0	4	

All-zero row

Auxiliary polynomial :

$$A(s) = s^2 + 4 \quad , \quad p = 2$$

$$\frac{dA(s)}{ds} = 2s + 0$$

Number of sign changes $= 0$, $n_+ = 0$

For the auxiliary equation $A(s) = 0$,

$P_+ = P_- = 0$ (No sign changes under the dotted line).

$P_o = p - 2P_+ = 2 - 2(0)$

$= 2$, $n_o = P_o = 2$

$n_- = n - (n_+ + n_o) = 3 - (0+2) = 1$

Root distribution : $\{1(-1), 2(0), 0(+)\}$

The system is marginally stable.

(e) $s^5 + 3s^4 + 2s^3 + 6s^2 + s + 3 = 0$

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Routh array

+	s^5	1	2	1	
	$\cancel{s^4}$	$\cancel{3}$	$\cancel{6}$	$\cancel{3}$	
+	s^4	1	2	1	
	$\cancel{s^3}$	$\cancel{0}$	$\cancel{0}$		All-zero row
	$\cancel{s^3}$	$\cancel{4}$	$\cancel{4}$		
+	s^3	1	1		
+	s^2	1	1		
	$\cancel{s^1}$	$\cancel{0}$			All-zero row
+	s^1	2			
+	s^0	1			

Auxiliary polynomial:

$(s+j1)^2(s-j1)^2$

$$A(s) = s^4 + 2s^2 + 1$$

$$\frac{dA(s)}{ds} = 4s^3 + 4s$$

Second Auxiliary polynomial:

$(s+j1)(s-j1)$

$$A'(s) = s^2 + 1$$

$$\frac{dA'(s)}{ds} = 2$$

No we have

$$n_+ = 0$$

$$P_+ = P_- = 0, \quad P_0 = n_0 = P - 2P_+ = 4 - 2(0) = 4$$

$(P_0' = P' = 2)$

$$n_- = n - (n_+ + n_0) = 5 - (4+0) = 1$$

Repeated roots on the $j\omega$-axis

Root distribution: $\{1(-),\ 4(0),\ 0(+)\}$

The system is unstable

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(f) $s^6 + s^5 + 3s^4 + 3s^3 + 3s^2 + 2s + 1 = 0$

Routh array

$+$	s^6	1	3	3	1
$+$	s^5	1	3	2	
	$\cancel{s^4}$	$\cancel{0}$	$\cancel{1}$	$\cancel{1}$	
$+$	s^4	ϵ	1	1	
$-$	s^3	$\dfrac{3\epsilon - 1}{\epsilon}$	$\dfrac{2\epsilon - 1}{\epsilon}$		
$+$	s^2	$\dfrac{-2\epsilon^2 + 4\epsilon - 1}{3\epsilon - 1}$	1		
	$\cancel{s^1}$	$\cancel{\dfrac{\epsilon(-4\epsilon + 1)}{-2\epsilon^2 + 4\epsilon - 1}} \to 0$		All-zero row	
$+$	s^1	2			
$+$	s^0	1			

Auxiliary polynomial, $A(s) = s^2 + 1$ (as $\epsilon \to 0$)

$$\frac{dA(s)}{ds} = 2$$

$n_+ = 2$

$P_+ = P_- = 0$, $\quad P_0 = n_0 = p - 2P_+ = 2$

$n_- = n - (n_+ + n_0) = 6 - (2+2) = 2$

Root distribution: $\{2(-1),\ 2(0),\ 2(+)\}$

The system is unstable.

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Problem 55

Characteristic equation, $1 + G(s) = 0$,

$$1 + \frac{K}{s(s+4)(s^2+s+1)} = 0$$

$$s(s+4)(s^2+s+1) + K = 0$$

$$s^4 + 5s^3 + 5s^2 + 4s + K = 0$$

Routh array

+	s^4	1	5	K
+	s^3	5	4	
+	s^2	$\frac{21}{5}$	K	
	s^1	$\dfrac{\frac{84}{5} - 5K}{21/5}$		
	s^0	K		

For the system to be stable,

$$\frac{84}{5} > 5K \quad \text{and} \quad K > 0$$

That is,

$$0 < K < \frac{84}{25}$$

or

$$0 < K < 3.36$$

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Problem 56

Characteristic equation, $1 + G(s) = 0$

$$1 + \frac{K}{(s+2)(s+4)(s^2+6s+25)} = 0$$

$$(s+2)(s+4)(s^2+6s+25) + K = 0$$

$$s^4 + 12s^3 + 69s^2 + 198s + 200 + K = 0$$

Routh array

+	s^4	1	69	$200 + K$
+	s^3	12	198	
+	s^2	52.5	$200 + K$	
	s^1	$\frac{7995 - 12K}{52.5}$		
	s^0	$200 + K$		

For the system to be stable,

$$K > -200 \quad \text{and} \quad 7995 - 12K > 0$$

That is,

$$-200 < K < \frac{7995}{12}$$

or

$$-200 < K < 666.25$$

The system is unstable when

$K > 666.25$ [redacted] or $K < -200$

The system is marginally stable when

$K = 666.25$ (sustained oscillations)

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where the auxiliary equation is

$$52.5\,s^2 + 200 + 666.25 = 0$$

$$s^2 = -\frac{866.25}{52.5} = -16.5$$

$$s = \pm j4.06$$

Frequency of oscillation, $\omega_n = 4.06$ rad/sec

Problem 57

Characteristic equation, $1 + G(s) = 0$,

$$1 + \frac{A(s+1)}{s^3 + Bs^2 + 2s + 1} = 0$$

$$s^3 + Bs^2 + (A+2)s + A + 1 = 0$$

Routh array

s^3	1	$A+2$
s^2	B	$A+1$
s^1	$\frac{B(A+2)-A-1}{B}$	
s^0	$A+1$	

All-zero row, $B \neq 0$

For sustained oscillations with frequency 2 rad/sec, we should have

$$B(A+2) - A - 1 = 0 \quad \text{(zero } s^1 \text{ row)}$$

and

$$Bs^2 + A + 1 = 0 \quad \text{(Auxiliary equation)}$$

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That is, from the auxiliary equation,

$$s^2 = \frac{-A-1}{B} = -4 \qquad (s = \pm j2)$$

$$A = 4B - 1$$

From the condition of zeroing the s^1 row,

$$B(4B-1+2) - 4B + 1 - 1 = 0 \qquad (B \neq 0)$$

$$B = 0.75$$

and

$$A = 4(0.75) - 1 = 2$$

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Sheet no. ( )

1. For each of The following Characteristic equations, determine The root distribution, i.e. The number of roots in The left half, right half, and on The imaginary axis of The s-plane. IS The Corresponding system Stable, Critical stable (marginally Stable) or unstable? why?

   a) $s^4+2s^3+s^2+4s+2=0$

   b) $s^5+s^4+2s^3+2s^2+3s+5=0$

   c) $s^5+3s^4+2s^3+6s^2+s+3=0$

   d) $s^5+2s^4+3s+6s^2+5s+3=0$

   e) $s^8+3s^7+10s^6+24s^5+48s^4+96s^3$
   $+128s^2+192s+128=0$

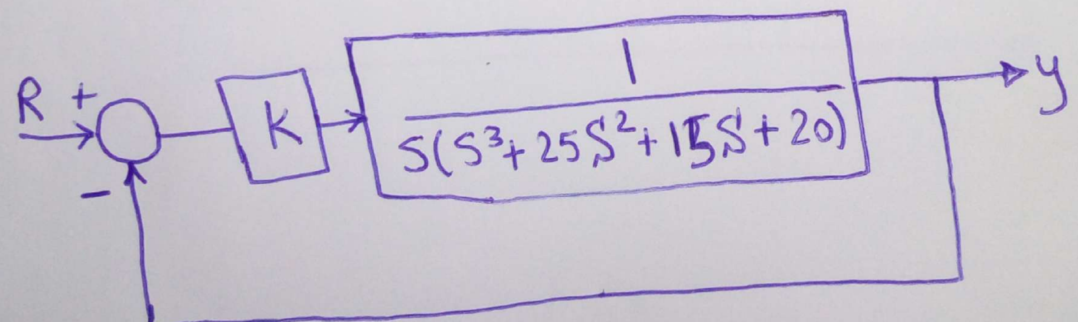
   f) $s^6+s^5+3s^4+3s^3+3s^2+2s+1=0$

   g) $s^3+2s^2+1=0$

   h) $s^4+2s^3+5s^2+10s=0$

2. For each of The following, Find The range of K For which The system is Stable.

   a)



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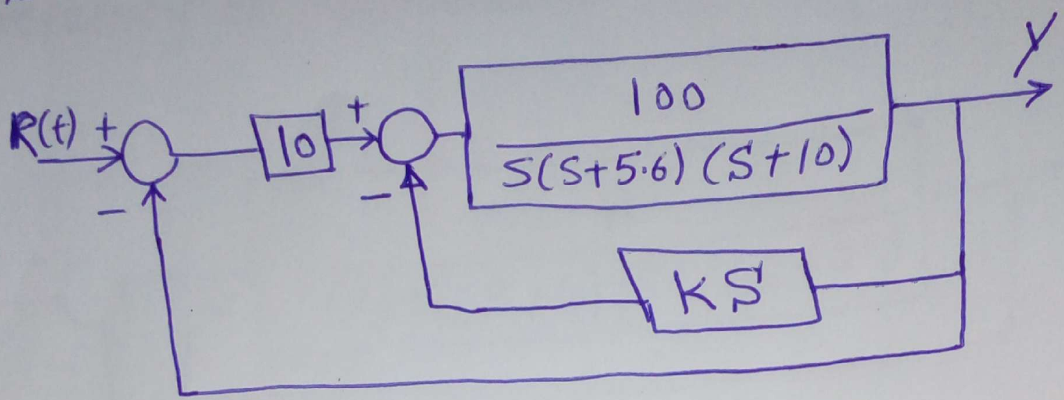
b) $S^4+S^3+S^2+3S+K=0$

c) $GH(S)=\dfrac{K}{S(S+1)(S+3)(S+4)}$

d) $S^3+(K+2)S^2+2KS+10=0$

e) $S^4+S^3+S^2+S+K=0$

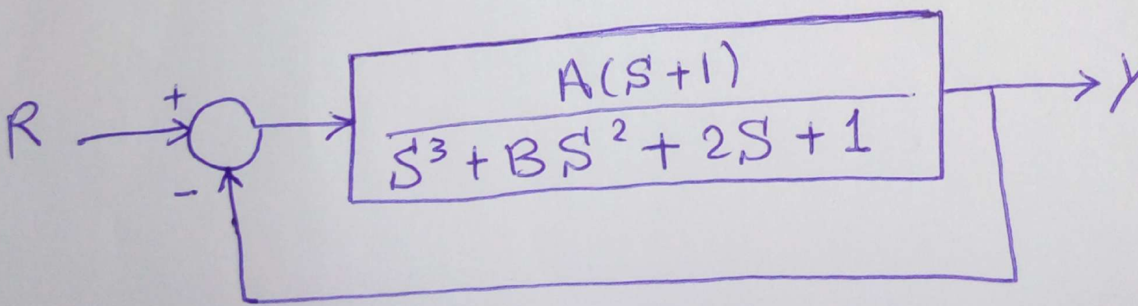
f)



g) $S^3+4KS^2+(K+5)S+10=0$

h) A unity feedback system has a forward transfer function $G(S)=\dfrac{K}{S(S+4)(S^2+S+1)}$

3] An oscillator system is shown in fig. Determine the values of A and B such that the system oscillates at a frequency of 2 rad/sec



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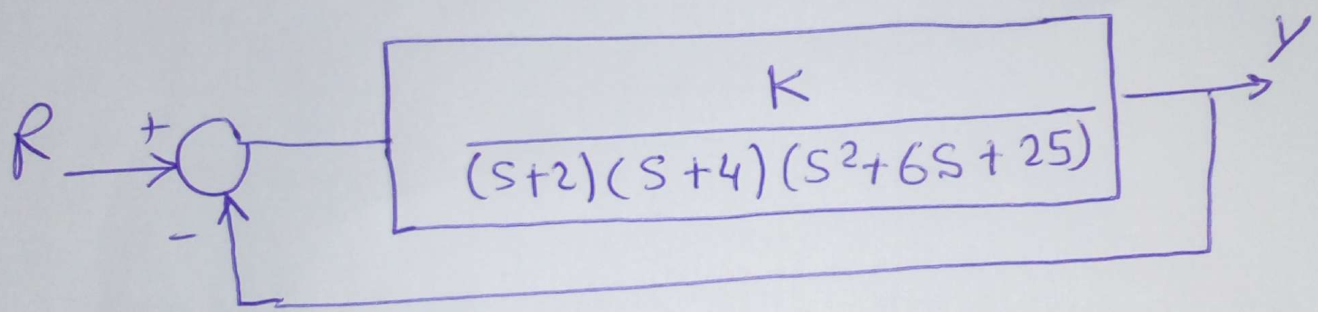
4 Consider The System shown in fig. By applying The Routh-Hurwitz criterion, discuss The stability of The System in terms of K.

Determine The Value of K which results in sustained oscillations. What is The corresponding frequency of oscillation?



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